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EXISTENCE OF NON-PREPERIODIC ALGEBRAIC POINTS FOR A RATIONAL SELF-MAP OF INFINITE ORDER

EKATERINA AMERIK

Let X be a smooth projective variety defined over a number field K and let $f : X \dashrightarrow X$ be a dominant rational self-map defined over the same number field. As shown in [AC], one can attach to f a dominant rational map $g : X \dashrightarrow T$, commuting with f and such that the fiber of g through a sufficiently general complex point $x \in X(\mathbb{C})$ is the Zariski closure of its iterated orbit (or "f-orbit") $\{f^k(x), k \in \mathbb{Z}_{\geq 0}\}$. Here "sufficiently general" means "outside of a countable union of proper subvarieties", and so this theorem does not give any information on the f-orbits of algebraic points, which, apriori, can have smaller Zariski closure than general complex points. Indeed, the field of algebraic numbers being countable, this countable union of proper subvarieties might contain all the algebraic points of X.

One would of course like to show that in reality this never happens and one can always find an algebraic point such that its f-orbit is "as large" as the general one. For instance, a conjecture already implicit in [AC] and formulated by Medvedev and Scanlon in [MS] (Conjecture 5.3) states that if no power of f preserves a non-trivial fibration, then there should be a point $x \in X(\bar{\mathbb{Q}})$ with Zariski-dense f-orbit ; a variant of this is an earlier conjecture by S.-W. Zhang stating the same in the case when f is regular and polarized (that is, there is an ample line bundle L on X with $f^*L = L^{\otimes q}$ for some $q > 1$).

What is certainly true in the case when f is regular and polarized is that, at least, there exist points in $X(\bar{\mathbb{Q}})$ with infinite f-orbits (that is, non-preperiodic algebraic points). The reason is that in this case, one can introduce the so-called *canonical height* $\hat{h}_L : X(\bar{\mathbb{Q}}) \to \mathbb{R}$ which is a Weil height function for L with the property $\hat{h}_L(f(x)) = q\hat{h}_L(x)$: it follows that the set of preperiodic points is a set of bounded height and therefore it cannot exhaust $X(\bar{\mathbb{Q}})$ (see [CS]). However, the examples of varieties admitting a regular polarized endomorphism are very scarce, so one would like to work in a more general setting: and the theory of canonical heights does not really work for a rational self-map f, even if there is an ample line bundle L such $f^*L = L^{\otimes q}$ with $q > 1$.

The purpose of this note is to provide a short proof of the existence of non-preperiodic algebraic points for arbitrary dominant rational self-maps of infinite order (Corollary 9), using, though, a result by E. Hrushovski which does not seem to have been treated in a very accessible way at the moment. The argument is very similar to the one used by Bell, Ghioca and Tucker to prove a version of the "dynamical Mordell-Lang conjecture" for étale endomorphisms of quasiprojective varieties: in fact this note is directly inspired by [BGT], and is, in some sense, a continuation of [ABR] where some general observations about iteration of algebraic points by a dominant

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ON AN AUTOMORPHISM OF Hilb[2] OF CERTAIN K3 SURFACES

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Abstract Following some remarks made by O'Grady and Oguiso, the potential density of rational points on the second punctual Hilbert scheme of certain K3 surfaces is proved.

Keywords: K3 surface; punctual Hilbert scheme; involution; rational points

2010 *Mathematics subject classification:* Primary 14J28; 14J50; 14G05

1. Introduction

In a recent note, Oguiso [**9**] uses the structure of the cohomology of compact hyperkähler manifolds [**12**] to describe the behaviour of the dynamical degrees of an automorphism of such a manifold and makes an explicit computation in some particular cases. In one of his examples he considers a K3 surface S admitting two embeddings as a quartic in $\mathbb{P}^3$ (given by two different very ample line bundles H_1 and H_2). Each embedding induces an involution of the second punctual Hilbert scheme (that is, the Hilbert scheme parametrizing finite subschemes of length 2) $X = \mathrm{Hilb}^{[2]}S = S^{[2]}$, where a pair of general points p_1, p_2 is sent to the complement of $\{p_1, p_2\}$ in the intersection of S and the line p_1p_2: it is shown in [**2**] that this involution is regular if and only if S does not contain lines. Oguiso considers the product of the two involutions and shows that this product is not induced from an automorphism of S, nor from any automorphism of a K3 surface S' such that $S'^{[2]} \cong S^{[2]}$.

On the other hand, in the recent past a few people have studied the potential density of rational points on K3 surfaces and their symmetric powers (see, for example, [**4**, **5**]). Recall that a variety X over a number field is called potentially dense if rational points on X become Zariski-dense after a finite field extension. In [**4**], it is proved that a K3 surface with an elliptic pencil or with infinitely many automorphisms is potentially dense. The proof proceeds by iterating rational curves by the automorphisms in the second case and by rational self-maps coming from the elliptic fibration (i.e. by fibrewise multiplication by a suitable number) in the first case. Several results concerning potential density on symmetric powers of K3 surfaces are given in [**5**]: the point to note here is that some

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Table of Contents - November 2011 - Volume 147, Issue 06

Research Article

Remarks on endomorphisms and rational points

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Abstract

Let X be an algebraic variety and let f:X--→X be a rational self-map with a fixed point q, where everything is defined over a number field K. We make some general remarks concerning the possibility of using the behaviour of f near q to produce many rational points on X. As an application, we give a simplified proof of the potential density of rational points on the variety of lines of a cubic fourfold, originally proved by Claire Voisin and the first author in 2007.

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2010 Mathematics Subject Classification

14G05 (primary); 11S82; 37P55 (secondary)

Keywords

rational points; rational maps; p-adic neighbourhood

CONSTRUCTING RATIONAL CURVES ON K3 SURFACES

FEDOR BOGOMOLOV, BRENDAN HASSETT, and YURI TSCHINKEL

Abstract

We develop a mixed-characteristic version of the Mori-Mukai technique for producin, rational curves on K3 surfaces. We reduce modulo p, produce rational curves on th resulting K3 surface over a finite field, and lift to characteristic zero. As an applicatior we prove that all complex K3 surfaces with Picard group generated by a class of degre two have an infinite number of rationalcurves.

1. Introduction

Let K be an algebraically closed field, and let S be a K3 surface defined over K It is known that S contains rational curves—see Mori and Mukai [18], as well a Theorem 7 and Proposition 17 below. In fact, an extension of the argument in [18 shows that the "general" K3 surface of given degree has infinitely many rational curve (see Theorem 9 and [7]). The idea is to specialize the K3 surface S to a K3 surface S with Picard group of rank 2, where some multiple of the polarization can be expresse as a sum of linearly independent classes of smooth rational curves. The union of thes rational curves deforms to an irreducible rational curve on S. This idea applies t K3 surfaces parameterized by points outside a countable union of subvarieties of th moduli space. In particular, a priori it does not apply to K3 surfaces over countabl fields, such as $\overline{\mathbb{F}}_p$ and $\overline{\mathbb{Q}}$. Of course, there are also other techniques proving densit of rational curves on special K3 surfaces, for example, certain Kummer surfaces (se [18]), surfaces with infinite automorphism groups (see [4, proof of Theorem 4.10] or with elliptic fibrations (see Remark 6 below). These K3 surfaces have Picard ran ≥ 2, and all except finitely many lattices in rank ≥ 3 correspond to K3 surfaces wit infinite automorphisms or elliptic fibrations (see [19], [28]).

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Current Developments in Algebraic Geometry
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Introduction to birational anabelian geometry

FEDOR BOGOMOLOV AND YURI TSCHINKEL

We survey recent developments in the Birational Anabelian Geometry program aimed at the reconstruction of function fields of algebraic varieties over algebraically closed fields from pieces of their absolute Galois groups.

Introduction

The essence of Galois theory is to *lose* information, by passing from a field k, an algebraic structure with two compatible operations, to a (profinite) group, its absolute Galois group G_k or some of its quotients. The original goal of testing solvability in radicals of polynomial equations in one variable over the rationals was superseded by the study of deeper connections between the arithmetic in k, its ring of integers, and its completions with respect to various valuations on the one hand, and (continuous) representations of G_k on the other hand. The discovered structures turned out to be extremely rich, and the effort led to the development of deep and fruitful theories: class field theory (the study of abelian extensions of k) and its nonabelian generalizations, the Langlands program. In fact, techniques from class field theory (Brauer groups) allowed one to deduce that Galois groups of global fields encode the field:

Keywords: Galois groups, function fields.

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VERSITA

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On the diffeomorphic type of the complement to a line arrangement in a projective plane

Research Article

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Abstract: We show that the diffeomorphic type of the complement to a line arrangement in a complex projective plane P^2 depends only on the graph of line intersections if no line in the arrangement contains more than two points in which at least two lines intersect. This result also holds for some special arrangements which do not satisfy this property. However it is not true in general, see [Rybnikov G., On the fundamental group of the complement of a complex hyperplane arrangement, Funct. Anal. Appl., 2011, 45(2), 137–148].

MSC: 14N20, 52C35, 32S22

Keywords: Line arrangement • Incidence matrix

Introduction

The study of topological and diffeomorphic type of the complement of a line arrangement in projective plane is one of the classical topics in complex algebraic geometry with many different results. See, for example, [1–6]. Note that if we have a deformation $\mathcal{L}_t$ of the line arrangement $\mathcal{L}$ in such a way that the multiplicities of intersection points remain the same during the deformation, then the diffeomorphic type of the complement $P^2 \setminus \mathcal{L}_t$ remains the same. In particular, we give a simple proof that two lines arrangements with the same intersection matrix deform in each other under assumption that

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• ARTICLES •

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Rationality of quotients by linear actions of affine groups

We feel honored to dedicate this article to our friend, colleague and teacher Fabrizio Catanese on the occasion of his 60th birthday

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Abstract Let $G = \mathrm{SL}_n(\mathbb{C}) \ltimes \mathbb{C}^n$ be the (special) affine group. In this paper we study the representation theory of G and in particular the question of rationality for V/G, where V is a generically free G-representation. We show that the answer to this question is positive (Theorem 6.1) if the dimension of V is sufficiently large and V is indecomposable. We explicitly characterize two-step extensions $0 \to S \to V \to Q \to 0$, with completely reducible S and Q, whose rationality cannot be obtained by the methods presented here (Theorem 5.3).

Keywords rationality, linear group quotients, affine groups

MSC(2000): 14E08, 14M20, 14L24

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1 Introduction

The well-known rationality problem in invariant theory asks whether V/G is always rational if V is a linear representation of a connected linear algebraic group G over $\mathbb{C}$. This seems to be extremely difficult, in general. However, it becomes a little more accessible if the unipotent radical of G is large in a certain sense, of which Miyata's theorem is the first example: if the action of G on V can be made triangular, then V/G is rational. We will give further evidence for the previous viewpoint in this paper by studying generically free quotients V/G, where $G = \mathrm{SL}_n(\mathbb{C}) \ltimes \mathbb{C}^n$ is the special affine group. In fact, if V is indecomposable and of sufficiently large dimension, these quotients are always rational, cf. Theorem 6.1 and Theorem 5.3 below. Some indecomposability assumption is really needed as there are families of decomposable generically free and arbitrarily large G-representations for which a proof of rationality amounts to proving stable rationality of level 1 for all generically free $\mathrm{SL}_n(\mathbb{C})$-representations, which is expected to be a hard problem, cf. Remark 5.4. One should also note that many rationality questions for reductive groups reduce to parabolic subgroups by the method of taking sections for the

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RECONSTRUCTION OF HIGHER-DIMENSIONAL FUNCTION FIELDS

FEDOR BOGOMOLOV AND YURI TSCHINKEL

ABSTRACT. We determine the function fields of varieties of dimension ≥ 2 defined over the algebraic closure of $\mathbb{F}_p$, modulo purely inseparable extensions, from the quotient by the second term in the lower central series of their pro-ℓ Galois groups.

2000 MATH. SUBJ. CLASS. 12F10, 14E08, 19C20, 19C30.

KEY WORDS AND PHRASES. Galois groups, function fields.

INTRODUCTION

Fix two distinct primes p and ℓ. Let $k = \overline{\mathbb{F}}_p$ be an algebraic closure of the finite field $\mathbb{F}_p$. Let X be an algebraic variety defined over k and $K = k(X)$ its function field. We will refer to X as a *model* of K; we will generally assume that X is normal and projective. Let $\mathcal{G}_K^a$ be the abelianization of the pro-ℓ-quotient $\mathcal{G}_K$ of the absolute Galois group of K. Under our assumptions on k, $\mathcal{G}_K^a$ is a torsion-free $\mathbb{Z}_\ell$-module isomorphic to $\mathbb{Z}_\ell^{\mathbb{N}}$. Let $\mathcal{G}_K^c$ be its canonical central extension — the second lower central series quotient of $\mathcal{G}_K$. It determines a set Σ_K of distinguished (primitive) finite-rank subgroups of $\mathcal{G}_K^a$: a topologically noncyclic subgroup σ lies in Σ_K if and only if

- the inverse image of σ in $\mathcal{G}_K^c$ is abelian;
- σ is maximal: there are no subgroups $\sigma' \subset \mathcal{G}_K^a$ whose preimages in $\mathcal{G}_K^c$ are abelian and which contain σ as a proper subgroup.

Our main theorem is

Theorem 1. *Let K and L be function fields over algebraic closures of finite fields k, resp. l, of characteristic $\neq \ell$. Assume that the transcendence degree of K over k is at least two and that there exists an isomorphism*

$$\Psi = \Psi_{K,L} \colon \mathcal{G}_K^a \xrightarrow{\sim} \mathcal{G}_L^a \tag{1.1}$$

of abelian pro-ℓ-groups inducing a bijection of sets Σ_K and Σ_L. Then k is isomorphic to l and there exists a constant $\epsilon \in \mathbb{Z}_\ell^*$ such that $\epsilon^{-1} \cdot \Psi$ is induced from a

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VERSITA

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Unramified cohomology of alternating groups

Research Article

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Abstract: We prove vanishing results for the unramified stable cohomology of alternating groups.

MSC: 14E08, 20J06, 20F36, 20B35

Keywords: Rationality • Alternating groups • Unramified cohomology

1. Introduction

Let k be an algebraically closed field, G a finite group, and V a faithful linear representation of G over k. The study of the fields of invariants $k(V)^G = k(V/G)$ for different groups G has provided with many examples of nonrational unirational varieties V/G and fields $k(V)^G$. Currently all the proofs of nonrationality for such quotients are based on nontriviality of the nonramified cohomology of G. These cohomology groups serve as simplest obstructions to the stable rationality of the field $k(V)^G$. They were introduced in [2, 7] and studied in [4–6]. In the case of degree 2 they appeared in the work of Saltman [12] under disguise of the Brauer group.

In [3, 4] the following conjecture was raised

Conjecture 1.1.
Let G be a finite simple group. Then

$$H^i_{k,\mathrm{un}}(G,\mathbb{Z}/\ell) = 0,$$

for all $i > 0$, all k and all ℓ.

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Р. Я. Будылин

Адельное построение класса Черна

Приводится формула, выражающая второй класс Черна $c_2(E)$ в терминах тривиализаций двумерного векторного расслоения E в схемных точках поверхности X над полем. Для этого по тривиализациям строится коцикл в адельном комплексе, связанном с пучком $\mathrm{K}_2(\mathscr{O}_X)$. Кроме того доказано, что формула Севери для второго класса Черна получается как частный случай формулы, построенной в этой работе.

Библиография: 10 названий.

Ключевые слова: класс Черна, адельный комплекс.

§ 1. Введение

В статье А. Н. Паршина [1] в 1983 г. появилась формула для класса Черна векторного расслоения на алгебраическом многообразии в терминах матриц перехода между тривиализациями расслоения в схемных точках многообразия.

Для произвольных обратимых матриц $X_1,\dots,X_m$ над коммутативной $\mathbb{Q}$-алгеброй A можно рассмотреть форму

$$p_m(X_1,\dots,X_m):=\operatorname{tr}(X_1^{-1}\cdots X_m^{-1}dX_1\wedge\cdots\wedge dX_m)\in\Omega_A^m.$$

Под внешним произведением матриц $X=(a_{ij})$ и $Y=(b_{ij})$ с коэффициентами в $\Omega^*(A)$ здесь подразумевается матрица $X\wedge Y:=\bigl(\sum_j a_{ij}\wedge b_{jl}\bigr)$. Напомним формулу Ньютона, выражающую элементарные симметрические функции σ_m через суммы степеней s_i:

$$\sigma_m=\frac{1}{m!}\begin{vmatrix} s_1 & 1 & 0 & \dots & 0\\ s_2 & s_1 & 2 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ s_{m-1} & s_{m-2} & s_{m-3} & \dots & m-1\\ s_m & s_{m-1} & s_{m-2} & \dots & s_1\end{vmatrix}.$$

Положим $w_A^m(X_1,\dots,X_m)$ равным значению элементарной симметрической функции σ_m после замены суммы степеней s_i в формуле Ньютона на $p_i(X_1,\dots,X_i)$, а умножения – на внешнее произведение. Пусть X – произвольная нётерова неприводимая схема размерности n конечного типа над полем нулевой характеристики. Пусть E – векторное расслоение на X, пучок $\mathscr{E}$ – локально свободный пучок $\mathscr{O}_X$-модулей, определяемый расслоением E. Для каждой схемной точки $\eta\in X$ рассмотрим пучковый слой $\mathscr{E}_\eta$. Пусть b_η – базис

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Неравенство, полученное при рассмотрении нестабильных решеток ранга 2

Р. Я. Будылин

Пусть K – числовое поле с кольцом целых F, $n = [K:\mathbb{Q}] < \infty$ и P – проективный модуль над кольцом F. Поля K_i – архимедовы пополнения поля K. Пусть r – число вещественных точек, s – комплексных. Мы будем пользоваться определениями из [1].

Определение 1. Структура F-решетки на модуле P есть набор скалярных (эрмитовых) произведений на каждом вещественном (комплексном) векторном пространстве $P\otimes_F K_i$. Если $K_i = \mathbb{C}$, то скалярное произведение предполагается эрмитовым.

Определение 2. Решетку называют полустабильной, если ее каноническая фильтрация имеет вид $0\subset P$. В противном случае решетка называется нестабильной.

Пусть $P = F^k$. Обозначим через $O(k,K_i)$ группу $O_k(\mathbb{R})$ для вещественных точек и группу $U_k(\mathbb{C})$ – для комплексных точек. Множество скалярных (эрмитовых) произведений над вещественным (комплексным) пополнением K_i отождествляется с $\mathrm{Gl}(k,K_i)/O(k,K_i)$. Обозначим группу $\prod\limits_{K_i}\mathrm{Gl}(k,K_i)$ через G, группу $\Bigl\{(x_i)\in G \mid \prod\limits_{K_i}\|x_i\| = 1\Bigr\}$ через G', группу $\mathrm{Gl}(k,F)$ через H, группу $\prod\limits_{K_i}O(k,K_i)$ через O. Множество решеток с точностью до эквивалентности отождествляется со множеством $H\backslash G/O$. Множество G/O отождествляется со множеством решеток с выделенным базисом в F^n. Назовем его элементы вложенными решетками. Элементы множества G'/O назовем специальными вложенными решетками. По теореме о сильной аппроксимации действие группы H на множестве G'/O – это действие с компактным фактором. Пусть множество M – фундаментальная область этого действия.

Пусть $N(k,K_i)$ – унипотентные матрицы из $\mathrm{Gl}(k,K_i)$, группа $A(k,K_i)$ – вещественные диагональные матрицы с положительными элементами на диагонали. Обозначим группу $\prod\limits_{K_i}A(k,K_i)$ через A, группу $\prod\limits_{K_i}N(k,K_i)$ через N. Разложение Ивасава дает разложение $G = ANO$. Пусть $da^{(i)}$ – мера Хаара на $A(k,K_i)$ равная $1/(a_1\cdots a_k)\,da_1\cdots da_k$, где a_i есть i-й диагональный элемент. После замены переменной $a_i = e^{e_i}$ получим $de_1\cdots de_k$. Пусть $dn^{(i)}$ – мера Хаара на $N(k,K_i)$, равная стандартной мере Лебега на $\mathbb{R}^{k(k-1)\dim(K_i)/2}$. Мера $dg^{(i)}$ на $\mathrm{Gl}(n,K_i)$, равная $\rho_{K_i}(a)\,da^{(i)}\,dn^{(i)}\,do^{(i)}$, где $\rho_{K_i}(a) = \prod\limits_{k<l}\Bigl(\dfrac{a_k}{a_l}\Bigr)^{\dim_{\mathbb{R}}(K_i)}$ и $do^{(i)}$ – произвольная мера Хаара на $O(K_i)$, является мерой Хаара [2; лемма 3]. Положим $\rho = \prod\limits_{K_i}\rho_{K_i}$, через dn и da обозначим меры Хаара на N и A соответственно. В разложении Ивасава для группы G' группа A заменяется на подгруппу $A'\subset A$, состоящую из наборов диагональных матриц (P^i) таких, что $\prod\limits_{K_i}\det(P^i)^{\dim_{\mathbb{R}}(K_i)} = 1$. Пусть da' – произвольная мера Хаара на A'.

Определение 3. Пусть X – пространство с левым действием группы G. Мера μ на X называется G-эквивариантной, если $g_*\mu = \mu$ для любого элемента $g\in G$.

Теорема 1. *Мера $\rho(a)\,da'\,dn$ является G'-эквивариантной на G'/O.*

Далее всюду $k = 2$. Пусть a, b – базис модуля F^2.

Работа выполнена при поддержке гранта МК-4881.2011.1, Лаборатории алгебраической геометрии ГУ-ВШЭ, гранта правительства РФ дог. 11.G34.31.0023, гранта РФФИ 11-01-00145а и программы "Ведущие научные школы" (грант НШ-4713.2010.1).

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A CR twistor space of a G_2-manifold

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Abstract

Let M be a G_2-manifold. We consider an almost CR-structure on the sphere bundle of unit tangent vectors on M, called *the CR twistor space*. This CR-structure is integrable if and only if M is a holonomy G_2-manifold. We interpret G_2-instanton bundles as CR-holomorphic bundles on its twistor space.

MSC: 53C25; 53C29; 53C15; 53C07

Keywords: G_2-manifold; Twistor space; CR-manifold; Calibration; Special holonomy

Article Outline

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Hodge theory on nearly Kähler manifolds

Misha Verbitsky

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ABSTRACT

Let (M, I, ω, Ω) be a nearly Kähler 6-manifold, that is, an $SU(3)$-manifold with -form Ω and Hermitian form ω which satisfies $d\omega = 3\lambda Re\Omega$, $dIm\Omega = -2\lambda$ nonzero real constant λ. We develop an analogue of the Kähler relations on proving several useful identities for various intrinsic Laplacians on M. When compact, these identities give powerful results about cohomology of M. We sh that harmonic forms on M admit a Hodge decomposition, and prove that H unless $p = q$ or $(p = 1, \quad q = 2)$ or $(p = 2, \quad q = 1)$.

KEYWORDS

nearly Kähler, G_2-manifold, Hodge decomposition, Hodge structure, Calabi-Yau manifold, almost complex structure, holonomy

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Hyperholomorphic connections on coherent sheaves and stability

Research Article

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Abstract: Let M be a hyperkähler manifold, and F a reflexive sheaf on M. Assume that F (away from its singularities) admits a connection ∇ with a curvature Θ which is invariant under the standard $SU(2)$-action on 2-forms. If Θ is square-integrable, such sheaf is called *hyperholomorphic*. Hyperholomorphic sheaves were studied at great length in [21]. Such sheaves are stable and their singular sets are hyperkähler subvarieties in M. In the present paper, we study sheaves admitting a connection with $SU(2)$-invariant curvature which is not necessary L^2-integrable. We show that such sheaves are polystable.

MSC: 14D21, 53C05, 53C07, 53C26, 53C28, 53C38, 53C55

Keywords: Hyperkahler manifold • Coherent sheaf • Stable bundle • Twistor space

1. Introduction

Yang–Mills theory of holomorphic vector bundles is one of the most spectacular successes of modern algebraic geometry. Developed by Narasimhan–Seshadri, Kobayashi, Hitchin, Donaldson, Uhlenbeck–Yau and others, this theory proved to be very fruitful in the study of stability and modular properties of holomorphic vector bundles. The Bogomolov–Miyaoka–Yau inequality and the Uhlenbeck–Yau theorem were used by Carlos Simpson in his groundbreaking work on variations of Hodge structures and flat bundles [15]. Later, it was shown [10, 20] that Yang–Mills approach is also useful in hyperkähler geometry and lends itself to an extensive study of stable bundles and their modular and twistor properties.

From an algebraic point of view, a coherent sheaf is a much more natural kind of object than a holomorphic vector bundle. This stresses the extreme importance of Bando–Siu theory [1] which extends Yang–Mills geometry to coherent sheaves.

In the present paper, we study the ramifications of Bando–Siu theory for hyperkähler manifolds.

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Manifolds with parallel differential forms and Kähler identities for G_2-manifolds

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Abstract

Let M be a compact Riemannian manifold equipped with a parallel differential form ω. We prove a version of the Kähler identities in this setting. This is used to show that the de Rham algebra of M is weakly equivalent to its subquotient $(H_c^*(M), d)$, called *the pseudo-cohomology* of M. When M is compact and Kähler, and ω is its Kähler form, $(H_c^*(M), d)$ is isomorphic to the cohomology algebra of M. This gives another proof of homotopy formality for Kähler manifolds, originally shown by Deligne, Griffiths, Morgan and Sullivan. We compute $H_c^*(M)$ for a compact G_2-manifold, showing that $H_c^i(M) \cong H^i(M)$ unless i=3,4. For i=3,4, we compute $H_c^*(M)$ explicitly in terms of the first-order differential operator $*d: \Lambda^3(M) \longrightarrow \Lambda^3(M)$.

Keywords: Rational homotopy; Formality; Kähler identities; G_2-manifolds

Article Outline

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Back to results | < Previous 2 of 2 next

Abstract References

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Moduli spaces of framed instanton bundles on CP^3 and twistor sections of moduli spaces of instantons on C^2

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Abstract

We show that the moduli space M of framed instanton bundles on CP^3 is isomorphic (as a complex manifold) to a subvariety in the moduli of rational curves of the twistor space of the moduli space of framed instantons on R^4. We then use this characterization to prove that M is equipped with a torsion-free affine connection with holonomy in $Sp(2n,C)$.

Keywords: Twistor theory; Moduli spaces of instantons

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OELJEKLAUS–TOMA MANIFOLDS ADMITTING NO COMPLEX SUBVARIETIES

Liviu Ornea and Misha Verbitsky

To Professor Vasile Brînzănescu at his sixty-fifth birthday

Abstract. The Oeljeklaus–Toma (OT)-manifolds are complex manifolds constructed by Oeljeklaus and Toma from certain number fields, and generalizing the Inoue surfaces S_m. On each OT-manifold we construct a holomorphic line bundle with semipositive curvature form ω_0 and trivial Chern class. Using this form, we prove that the OT-manifolds admitting a locally conformally Kähler structure have no non-trivial complex subvarieties. The proof is based on the Strong Approximation theorem for number fields, which implies that any leaf of the null-foliation of ω_0 is Zariski dense.

Contents

1. Introduction

1.1. OT-manifolds and their subvarieties. The Oeljeklaus–Toma (OT)-manifolds are an important class of compact complex manifolds not admitting a Kähler metric. They were discovered by Oeljeklaus and Toma in 2005 [5]. The construction of OT-manifolds uses the Dirichlet unit theorem from number theory (Subsection 1.2; see [7] for additional details of this construction and many related questions). Starting from a degree 3 number field, one obtains a two-dimensional OT-manifold known as Inoue surface S_m (see [2]).

For some number fields, the OT-manifolds are locally conformally Kähler (LCK). An LCK structure on a complex manifold is a Kähler metric on its universal cover $\tilde{M}$, such that the deck transform maps act on $\tilde{M}$ by homotheties. The OT-manifolds serve an important function in the theory of LCK manifolds, providing a counterexample

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Key words and phrases. Locally conformally Kähler manifold, Kähler potential, positive bundle, complex subvariety, Inoue surface.

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Р. А. Девятов

Смежностные многогранники с небольшим числом вершин

Построена серия смежностных многогранников в $\mathbb{R}^{2d}$ с числом вершин $N = 2d + 4$. Все эти многогранники имеют плоскую диаграмму Гейла определенного вида, а именно, в ней ровно $d + 3$ черных точек, лежащих в выпуклом положении. Такие диаграммы Гейла параметризуются 3-деревьями (деревьями с некоторой дополнительной структурой). Для всех построенных многогранников число граней размерности m, содержащих заданную вершину A, зависит лишь от d и m.

Библиография: 7 названий.

Ключевые слова: комбинаторика многогранников, комбинаторика набора точек, смежностные многогранники, диаграммы Гейла.

§ 1. Введение

Выпуклый многогранник в пространстве $\mathbb{R}^D$ называется *смежностным*, если любые $\lfloor D/2 \rfloor$ его вершин образуют грань. Иногда *k-смежностным многогранником* называют многогранник, у которого любые k вершин образуют грань. В смысле этого определения мы рассматриваем только $\lfloor D/2 \rfloor$-смежностные многогранники размерности D. Мы будем рассматривать случай, когда D четно ($D = 2d$).

Примером смежностного многогранника является циклический многогранник (см. [1]). Для его построения используется кривая моментов в $\mathbb{R}^D$, которая содержит все точки вида

$$(t, t^2, t^3, \dots, t^D), \qquad t \in \mathbb{R}.$$

Выпуклая оболочка любого конечного подмножества кривой моментов называется *циклическим многогранником*. Циклический многогранник является смежностным.

Известно, что число граней циклического многогранника фиксированной размерности является максимально возможным для многогранников такой размерности с таким количеством вершин (см. [1], [2]). В частности, если размерность многогранника равна $D = 2d$, а число вершин равно $D + 4$, то число его не-граней (т.е. наборов вершин, не являющихся гранями) в размерности $D + 3 - k$ равно $(d + 2)^2 C_{d+1}^{k-2} + 2C_{d+2}^{k}$. Смежностные многогранники комбинаторно жесткие (см. [3]), т.е. комбинаторный тип смежностного многогранника определяет комбинаторику конфигурации вершин.

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А. Д. Елагин

Когомологическая теория спуска для морфизма стеков и для эквивариантных производных категорий

В работе найдены необходимые и достаточные условия, при которых для морфизма $X \to S$ алгебраических многообразий (или, в более общем случае, стеков) производная категория S восстанавливается методами теории спуска по производной категории X. Показано, что в случае действия линейно редуктивной алгебраической группы G на схеме X из этого результата следует эквивалентность производной категории G-эквивариантных пучков на X и категории объектов в производной категории пучков на X с заданным на них действием G.

Библиография: 18 названий.

Ключевые слова: производные категории, теория спуска, алгебраическое многообразие.

§ 1. Введение

Хорошо известно, что пучки на многообразии можно определять локально. А именно, пусть $S = \bigcup U_i$ – покрытие многообразия S открытыми подмножествами. Задание пучка на S равносильно заданию семейства пучков F_i на U_i и изоморфизмов склейки $\varphi_{ij}: F_i|_{U_i\cap U_j} \to F_j|_{U_i\cap U_j}$, удовлетворяющих условию коцикла: на пересечениях $U_i\cap U_j\cap U_k$ выполнено $\varphi_{ik} = \varphi_{jk}\circ\varphi_{ij}$. Можно задавать пучки и при помощи покрытий более широкого класса. Пусть $p\colon X \to S$ – накрытие (например, накрытие топологических пространств или конечный плоский морфизм схем), p_i и p_{ij} – проекции расслоенных произведений $X \times_S X$ и $X \times_S X \times_S X$ на сомножители. Тогда задание пучка на базе S равносильно заданию пучка F на X вместе с изоморфизмом склейки пучков $\theta\colon p_1^*F \to p_2^*F$ на $X\times_S X$, удовлетворяющим условию коцикла: изоморфизмы $p_{13}^*\theta$ и $p_{23}^*\theta\circ p_{12}^*\theta$ на $X \times_S X \times_S X$ равны.

Заметим, что первое утверждение про открытое покрытие – по сути, частный случай второго для $X = \bigsqcup U_i$.

Возникает естественный вопрос: будут ли верны подобные утверждения, если заменить пучки на объекты производной категории когерентных пучков на алгебраическом многообразии?

Работа выполнена при поддержке Российского фонда фундаментальных исследований (гранты № 09-01-00242-а, № 10-01-93110-НЦНИЛ-а, № 10-01-93113-НЦНИЛ-а), гранта Президента РФ МД-2712.2009.1, а также Фонда поддержки молодых ученых "Конкурс Мёбиуса", Научного фонда ГУ-ВШЭ "Управление поддержки академических исследований" (грант № 10-09-0015), Программы Президента РФ поддержки ведущих научных школ РФ (грант № НШ-4713.2010.1).

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Elagin, A. D. (RS-AOS-IT)

Cosmological descent theory for a morphism of stacks and for equivariant derived categories. 1468-4802

(Russian. Russian summary)

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Deformation theory of objects in homotopy and derived categories III: Abelian categories ☆

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Abstract

This is the third paper in a series. In Part I we developed a deformation theory of objects in homotopy and derived categories of DG categories. Here we show how this theory can be used to study deformations of objects in homotopy and derived categories of abelian categories. Then we consider examples from (noncommutative) algebraic geometry. In particular, we study noncommutative Grassmanians that are true noncommutative moduli spaces of structure sheaves of projective subspaces in projective spaces.

Keywords: Deformation theory; Derived categories; Noncommutative geometry

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УДК 512.66

А. И. Ефимов

Доказательство гипотезы Концевича–Сойбельмана

Хорошо известно, что "категория Фукаи" является на самом деле A_∞-предкатегорией в смысле М. Концевича и Я. Сойбельмана. Это связано с тем, что, вообще говоря, пространства морфизмов определены только для трансверсальных пар лагранжевых подмногообразий, а высшие умножения определены только для трансверсальных последовательностей лагранжевых подмногообразий. Концевич и Сойбельман сформулировали следующую гипотезу: для любого градуированного коммутативного кольца k классы квазиэквивалентности A_∞-предкатегорий над k находятся в биекции с классами квазиэквивалентности A_∞-категорий над k с сильными (или слабыми) тождественными морфизмами.

В работе эта гипотеза доказана для существенно малых A_∞-(пред)категорий в случае, когда k является полем. В частности, отсюда следует, что A_∞-предкатегорию Фукаи можно заменить на квазиэквивалентную настоящую A_∞-категорию.

Кроме того, представлена естественная конструкция предтриангулированной оболочки для A_∞-предкатегорий и доказана ее инвариантность относительно квазиэквивалентностей.

Библиография: 8 названий.

Ключевые слова: A_∞-категории, категории Фукаи, гомологическая зеркальная симметрия.

§ 1. Введение

Замечательная конструкция К. Фукаи [1] связывает с симплектическим многообразием ($\mathbb{Z}$- или $\mathbb{Z}/2$-)градуированную A_∞-предкатегорию в смысле М. Концевича и Я. Сойбельмана [2]. Ее объекты – это лагранжевы подмногообразия с некоторыми дополнительными структурами.

Это – не настоящая A_∞-категория, так как, вообще говоря, пространства морфизмов определены только для трансверсальных пар лагранжевых подмногообразий, а высшие умножения определены только для трансверсальных последовательностей лагранжевых подмногообразий. Конструкция Фукаи используется в категорной интерпретации зеркальной симметрии [3] для многообразий Калаби–Яу и в дальнейших обобщениях на случаи многообразий Фано и многообразий общего типа. Этот подход к зеркальной симметрии известен как гипотеза Концевича о гомологической зеркальной симметрии. Различные версии и аспекты A_∞-предкатегорий Фукаи систематически изложены в книге [4].

Работа выполнена при финансовой поддержке Российского фонда фундаментальных исследований (грант № 4713.2010.1), Фонда "Династия" и Лаборатории алгебраической геометрии и ее приложений Высшей школы экономики (грант Правительства РФ № 11.G34.31.0023).

ASYMPTOTIC METHODS IN NUMBER THEORY AND ALGEBRAIC GEOMETRY

by

Philippe Lebacque & Alexey Zykin

Abstract. — The paper is a survey of recent developments in the asymptotic theory of global fields and varieties over them. First, we give a detailed motivated introduction to the asymptotic theory of global fields which is already well shaped as a subject. Second, we treat in a more sketchy way the higher dimensional theory where much less is known and many new research directions are available.

Résumé. — Cet article est un survol des développements récents dans la théorie asymptotique des corps globaux et des variétés algébriques définies sur les corps globaux. Dans un premier temps, nous donnons une introduction détaillée et motivée à la théorie asymptotique des corps globaux, théorie déjà bien établie. Puis nous aborderons plus rapidement la théorie asymptotique en dimension supérieure où peu de choses sont connues et où bien des directions de recherche sont ouvertes.

1. Introduction: the origin of the asymptotic theory of global fields

The goal of this article is to give a survey of asymptotic methods in number theory and algebraic geometry developed in the last decades. The problems that are treated by the asymptotic theory of global fields (that is number fields or function fields) and varieties over them are quite diverse in nature. However, they are connected by the use of zeta functions, which play the key role in the asymptotic theory.

We begin by a very well known problem which lies at the origin of the asymptotic theory of global fields. Let $\mathbb{F}_r$ be the finite field with r elements. For a smooth projective curve C over $\mathbb{F}_r$ we let $N_r(C)$ be the number of $\mathbb{F}_r$-point on C. We denote by $g(C)$ be the genus of C.

2000 ***Mathematics Subject Classification.*** — 11R42,11R29,11G40.

Key words and phrases. — Towers of global fields, L-functions in family, Brauer–Siegel theorem.

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Schubert calculus for algebraic cobordism

By *Jens Hornbostel* at Wuppertal and *Valentina Kiritchenko* at Moscow

Abstract. We establish a Schubert calculus for Bott–Samelson resolutions in the algebraic cobordism ring of a complete flag variety G/B extending the results of Bressler–Evens [4] to the algebro-geometric setting.

1. Introduction

We fix a base field k of characteristic 0. Algebraic cobordism $\Omega^*(-)$ has been invented some years ago by Levine and Morel [14] as the universal oriented algebraic cohomology theory on smooth varieties over k. In particular, its coefficient ring $\Omega^*(k)$ is isomorphic to the Lazard ring $\mathbb{L}$ (introduced in [12]). In a recent article [15], Levine and Pandharipande show that algebraic cobordism $\Omega^*(X)$ allows a presentation with generators being projective morphisms $Y \to X$ of relative codimension $n (= \dim(X) - \dim(Y))$ between smooth varieties and relations given by a refinement of the naive algebraic cobordism relation (involving double point relations). A recent result of Levine [13] which relies on unpublished work of Hopkins and Morel asserts an isomorphism $\Omega^*(-) \cong MGL^{2*,*}(-)$ between Levine–Morel and Voevodsky algebraic cobordism for smooth quasiprojective varieties. In particular, algebraic cobordism is representable in the motivic stable homotopy category.

In short, algebraic cobordism is to algebraic varieties what complex cobordism $MU^*(-)$ is to topological manifolds.

The above fundamental results being established, it is high time for computations, which have been carried out only in a very small number of cases (see e.g. [22] and [23]). The present article focuses on cellular varieties X, for which the additive structure of $\Omega^*(X)$ is easy to describe: it is the free $\mathbb{L}$-module generated by the cells (see the next section for more precise definitions, statements, proofs and references). So, additively, algebraic cobordism for cellular varieties behaves exactly as Chow groups do. Of course, algebraic K-theory also behaves in a similar way, but we will restrict our comparisons here and

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Abstract

We establish a Schubert calculus for Bott–Samelson resolutions in the algebraic cobordism ring of a complete flag variety *G/B* extending the results of Bressler–Evens [Trans. Amer. Math. Soc. 331: 799–813, 1992] to the algebro-geometric setting.

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Table of Contents - 01 May 2011 - Volume 147, Issue 03

Research Article

Base change for semiorthogonal decompositions

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Abstract

Let X be an algebraic variety over a base scheme S and $\phi: T \to S$ a base change. Given an admissible subcategory $\mathcal{A}$ in $\mathcal{D}^b(X)$, the bounded derived category of coherent sheaves on X, we construct under some technical conditions an admissible subcategory $\mathcal{A}_T$ in $\mathcal{D}^b(X \times_S T)$, called the base change of $\mathcal{A}$, in such a way that the following base change theorem holds: if a semiorthogonal decomposition of $\mathcal{D}^b(X)$ is given, then the base changes of its components form a semiorthogonal decomposition of $\mathcal{D}^b(X \times_S T)$. As an intermediate step, we construct a compatible system of semiorthogonal decompositions of the unbounded derived category of quasicoherent sheaves on X and of the category of perfect complexes on X. As an application, we prove that the projection functors of a semiorthogonal decomposition are kernel functors.

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base change; semiorthogonal decomposition

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УДК 512.53+512.544+512.772.5

Вик. С. Куликов

Полугруппы разложений на множители и неприводимые компоненты пространства Гурвица

Определена естественная структура полугруппы (изоморфной полугруппе разложений на множители единицы в симметрической группе) на множестве неприводимых компонент пространства Гурвица отмеченных накрытий степени d проективной прямой $\mathbb{P}^1$ с фиксированными типами ветвлений. Доказано, что эта полугруппа является конечно определенной. Исследована также проблема, когда наборы типов ветвления однозначно определяют соответствующие неприводимые компоненты пространства Гурвица. В частности, полностью описано множество неприводимых компонент пространства Гурвица трехлистных накрытий проективной прямой.

Библиография: 14 наименований.

Ключевые слова: полугруппа, разложения на множители элемента группы, неприводимые компоненты пространства Гурвица.

Введение

Обычно, чтобы исследовать пространство Гурвица $\mathrm{HUR}_d(\mathbb{P}^1)$ накрытий проективной прямой $\mathbb{P}^1 := \mathbb{CP}^1$ степени d, используется следующий подход. Фиксируются группа Галуа G накрытий, число b точек ветвления и типы локальных монодромий ветвления (т.е. фиксируются наборы, состоящие из b классов сопряженности элементов группы G), и после этого множество наборов, состоящих из представителей этих классов сопряженности, исследуется с точностью до так называемых преобразований Гурвица (см., например, [1]–[9]). Существует несколько других проблем (например, описать множество плоских алгебраических кривых с точностью до эквисингулярной деформации, или, более общо, описать множество плоских псевдоголоморфных кривых с точностью до симплектической деформации, описать множество симплектических пучков Лефшеца с точностью до диффеоморфизмов), в которых также возникают аналогичные объекты, а именно конечные наборы, состоящие из элементов некоторых групп и рассматриваемые с точностью до преобразований Гурвица (см., например, [10]–[12]). (В случае плоских алгебраических и пседоголоморфных кривых, чтобы получить такие наборы, выбирается пучок прямых (псевдопрямых), задающий расслоение над проективной прямой $\mathbb{P}^1$.) Как было показано в [13], на множествах таких наборов, рассматриваемых с точностью до преобразований Гурвица, имеются естественные структуры полугрупп, а именно

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Полугруппы разложений на множители и неприводимые компоненты пространства Гурвица. II

Вик. С. Куликов
Математический институт им. В. А. Стеклова РАН

Данная статья является продолжением статьи [?]. Рассматривается пространство Гурвица $\mathrm{HUR}^G_{d,t}(\mathbb{P}^1)$ накрытий степени d проективной прямой $\mathbb{P}^1$ с группой Галуа G, и имеющих фиксированный тип монодромии t, состоящий из набора локальных типов монодромий (т.е. набора классов сопряженности перестановок σ из симметрической группы S_d, которая действует на множестве $I_d = \{1, \dots, d\}$). Мы доказываем, что если тип t содержит достаточно большое число локальных монодромий, принадлежащих классу сопряженности C нечетной перестановки σ, которая оставляет неподвижными $f_C \geqslant 2$ элементов из I_d, то пространство Гурвица $\mathrm{HUR}^{S_d}_{d,t}(\mathbb{P}^1)$ является неприводимым многообразием.

полугруппа, разложения на множители элемента группы, неприводимые компоненты пространства Гурвица.

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УДК 512.743

И. В. Нетай

Параболически связные подгруппы

Найдены все редуктивные сферические подгруппы группы $\mathrm{SL}(n)$, для которых пересечения с каждой параболической подгруппой группы $\mathrm{SL}(n)$ связны. Это условие гарантирует алгебраичность открытых эквивариантных вложений соответствующих однородных пространств в пространства Мойшезона.

Библиография: 6 названий.

Ключевые слова: редуктивная группа, параболическая подгруппа, сферическая подгруппа, флаг, пространство Мойшезона.

§ 1. Введение

Пусть G – связная редуктивная алгебраическая группа над полем комплексных чисел $\mathbb{C}$.

Определение 1. Замкнутую подгруппу $H\subseteq G$ назовем *параболически связной*, если для любой параболической подгруппы $P\subseteq G$ пересечение $P\cap H$ связно.

Полезно отметить, что если для данной алгебраической подгруппы H ее пересечение с любой борелевской подгруппой $B\subset G$ связно, то H параболически связна в G. В самом деле, пусть $P\subseteq G$ – параболическая подгруппа и $B\subseteq G$ – борелевская подгруппа, содержащаяся в P. Тогда B также является борелевской подгруппой в P. Всякий элемент связной алгебраической подгруппы P лежит в некоторой ее борелевской подгруппе (см. [1; гл. 8, § 22]), и $H\cap P=\bigcup_{B\subseteq P}(H\cap B)$. В этом объединении каждый элемент $H\cap B$ связен и содержит единицу, так что $H\cap P$ связно.

Поскольку подгруппа унипотентной группы связна, каждая унипотентная подгруппа $H\subset G$ является параболически связной. В работе [2] показано, что для связной редуктивной группы H диагональ $\Delta H=\{(h,h):h\in H\}$ параболически связна в группе $G=H\times H$ (теорема 3).

Напомним, что алгебраическая подгруппа $H\subseteq G$ называется *сферической*, если индуцированное действие борелевской подгруппы B группы G на однородном пространстве G/H имеет открытую орбиту. Основным результатом настоящей работы является классификация параболически связных редуктивных сферических подгрупп в группе $\mathrm{SL}(n)$. Наша задача – выбрать параболически связные подгруппы из списка связных редуктивных сферических подгрупп, полученного в работе [3]. Символом $\mathrm{S}(\mathrm{GL}(m)\times\mathrm{GL}(n))$ обозначим подгруппу в $\mathrm{SL}(m+n)$, состоящую из всех блочнодиагональных матриц, размеры блоков которых равны m и n. Группа $\mathrm{SL}(m)\times\mathrm{SL}(n)$ вложена в группу $\mathrm{SL}(m+n)$

Abstract References

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Formal completions and idempotent completions of triangulated categories of singularities☆

Dedicated to the blessed memory of my adviser Vasily Alexeevich Iskovskikh

Dmitri Orlov

Algebra Section, Steklov Mathematical Institute RAN, Gubkin str. 8, Moscow 119991, Russia

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Abstract

The main goal of this paper is to prove that the idempotent completions of triangulated categories of singularities of two schemes are equivalent if the formal completions of these schemes along singularities are isomorphic. We also discuss Thomason's theorem on dense subcategories and a relation to the negative K-theory.

Keywords: Triangulated categories of singularities; Idempotent completion

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Matrix factorizations for nonaffine LG-models

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Mathematische Annalen

Matrix factorizations for nonaffine LG-models

Dmitri Orlov

Received: 21 February 2011 / Revised: 9 April 2011

Abstract We propose a natural definition of a category of matrix factorizations for nonaffine Landau–Ginzburg models. For any LG-model we construct a fully faithful functor from the category of matrix factorizations defined in this way to the triangulated category of singularities of the corresponding fiber. We also show that this functor is an equivalence if the total space of the LG-model is smooth.

1 Introduction

In the paper [8] we established a connection between categories of D-branes of type B (B-branes) in affine Landau–Ginzburg models and triangulated categories of singularities of the singular fibers. A mathematical definition for the category of B-branes in affine Landau–Ginzburg models was proposed by M. Kontsevich. According to his proposal the superpotential W will deform complexes of coherent sheaves to "W-twisted" complexes. These are 2-periodic chains of maps of vector bundles in which the composition of two consequtive maps is no longer required to be zero, but instead is equal to multiplication by W. Such chains are called matrix factorizations of W and so Kontsevich predicted that the category of B-branes should be the category of matrix factorizations. Kapustin and Li verified [4] the equivalence of this definition with the physics notion of B-branes in LG-models in the case of the usual quadratic

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COHERENT ANALOGUES OF MATRIX FACTORIZATIONS AND RELATIVE SINGULARITY CATEGORIES

LEONID POSITSELSKI

ABSTRACT. We define the triangulated category of relative singularities of a closed subscheme in a scheme. When the closed subscheme is a Cartier divisor, we consider matrix factorizations of the related section of a line bundle, and their analogues with locally free sheaves replaced by coherent ones. The appropriate exotic derived category of coherent matrix factorizations is then identified with the triangulated category of relative singularities, while the similar exotic derived category of locally free matrix factorizations is its full subcategory. The latter category is identified with the kernel of the direct image functor corresponding to the closed embedding of the zero locus and acting between the conventional (absolute) triangulated categories of singularities. Similar results are obtained for matrix factorizations of infinite rank; and two different "large" versions of the triangulated category of singularities, due to Orlov and Krause, are identified in the case of a divisor in a smooth scheme. Contravariant (coherent) and covariant (quasi-coherent) versions of the Serre–Grothendieck duality theorems for matrix factorizations are established, and pull-backs and push-forwards of matrix factorizations are discussed at length. A number of general results about derived categories of the second kind for CDG-modules over quasi-coherent CDG-algebras are proven on the way.

CONTENTS

MOSCOW MATHEMATICAL JOURNAL
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MIXED ARTIN–TATE MOTIVES WITH FINITE COEFFICIENTS

LEONID POSITSELSKI

ABSTRACT. The goal of this paper is to give an explicit description of the triangulated categories of Tate and Artin–Tate motives with finite coefficients $\mathbb{Z}/m$ over a field K containing a primitive m-root of unity as the derived categories of exact categories of filtered modules over the absolute Galois group of K with certain restrictions on the successive quotients. This description is conditional upon (and its validity is equivalent to) certain Koszulity hypotheses about the Milnor K-theory/Galois cohomology of K. This paper also purports to explain what it means for an arbitrary nonnegatively graded ring to be Koszul. Tate motives with integral coefficients are discussed in the "Conclusions" section.

2010 MATHEMATICS SUBJECT CLASSIFICATION. 12G05, 19D45, 16S37.

KEY WORDS AND PHRASES. Tate motives, Artin–Tate motives, motives with finite coefficients, Milnor–Bloch–Kato conjecture, Beilinson–Lichtenbaum conjecture, $K(\pi, 1)$ conjecture, Koszulity conjecture, Koszul algebras, nonflat Koszul rings, silly filtrations.

CONTENTS

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Two kinds of derived categories, Koszul duality, and comodule-contramodule correspondence

Author(s): Leonid Positselski
Journal: Memoirs of the AMS **212** (2011), no. 996.
MSC (2010): Primary 18E30, 18G10, 16T15, 16S37, 14F10; Secondary 18G55, 17B55, 16E65, 18G15, 58J10
Posted: November 19, 2010
Retrieve article in: PDF

Abstract: The aim of this paper is to construct the derived nonhomogeneous Koszul duality. We consider the derived categories of DG-modules, DG-comodules, and DG-contramodules, the coderived and contraderived categories of CDG-modules, the coderived category of CDG-comodules, and the contraderived category of CDG-contramodules. The equivalence between the latter two categories (the comodule-contramodule correspondence) is established. Nonhomogeneous Koszul duality or ``triality" (an equivalence between exotic derived categories corresponding to Koszul dual (C)DG-algebra and CDG-coalgebra) is obtained in the conilpotent and nonconilpotent versions. Various A-infinity structures are considered, and a number of model category structures are described. Homogeneous Koszul duality and D-Ω duality are discussed in the appendices.

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VERSITA

Central European Journal of Mathematics

Hori–Vafa mirror models for complete intersections in weighted projective spaces and weak Landau–Ginzburg models

Research Article

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Abstract: We prove that Hori–Vafa mirror models for smooth Fano complete intersections in weighted projective spaces admit an interpretation as Laurent polynomials.

MSC: 14J33, 14M10, 14Q15

Keywords: Hori–Vafa models • Landau–Ginzburg models • Complete intersections

Mirror symmetry of variations of Hodge structures states that for any smooth Fano variety X there exists a dual Landau–Ginzburg model $f\colon Y \to \mathbb{C}$ such that an essential part of the regularized quantum differential equation for X is of Picard–Fuchs type. In other words, the solutions of a certain differential equation (constructed via genus 0 Gromov–Witten invariants for X — the numbers which count rational curves lying on X) are the periods of the dual family (for more details and references see [10]). By definition, the relevant Picard–Fuchs differential equation depends only on relative birational type of Y. If one assumes $Y = (\mathbb{C}^*)^N$ one can translate mirror correspondence to the quantitative level, that is, to combinatorics of Laurent polynomials. Then f may be represented by a Laurent polynomial, which is called a (very) weak Landau–Ginzburg model. The following conjecture states that this hypothesis is not very restrictive, particulary for the case of $\operatorname{Pic} X = \mathbb{Z}$.

Conjecture 0.1 ([10]).
Any smooth Fano variety of dimension N with Picard rank 1 has a weak Landau–Ginzburg model $f \in \mathbb{C}[x_1^{\pm 1}, \ldots, x_N^{\pm 1}]$.

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Group Actions on Affine Cones

Takashi Kishimoto, Yuri Prokhorov, and Mikhail Zaidenberg

To Peter Russell on the occasion of his 70th birthday

ABSTRACT. We address the following question:

For which smooth projective varieties the corresponding affine cone admits an action of a connected algebraic group different from the standard $\mathbb{G}_m$-action by scalar matrices and its inverse action?

We show in particular that the affine cones over anticanonically embedded smooth del Pezzo surfaces of degree ≥ 4 possess such an action. Besides, we give some examples of rational Fano threefolds which have this property. A question in [29] whether this property holds also for smooth cubic surfaces in $\mathbb{P}^3$, occurs to be out of reach for our methods. We provide nevertheless a general geometric criterion that could be helpful in this case as well.

Introduction

All varieties in this paper are defined over $\mathbb{C}$. By Corollary 1.13 in [29], an isolated Cohen–Macaulay singularity (X, x) of a normal quasiprojective variety X is rational provided that X admits an effective action of the additive group $\mathbb{G}_a = \mathbb{G}_a(\mathbb{C}) = \mathbb{C}_+$, in particular of a connected nonabelian algebraic group. In the opposite direction, let us observe that, for instance, the singularity at the origin of the affine Fermat cubic in $\mathbb{A}^4$

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 = 0$$

is rational. The question was raised [29, Question 2.22] whether it also admits a nondiagonal action of a connected algebraic group, in particular, a $\mathbb{G}_a$-action.

2010 *Mathematics Subject Classification.* Primary 14R20, 14J45; Secondary 14J50, 14R05.

Key words and phrases. Affine cone, del Pezzo surface, Fano threefold, automorphism, additive gronp, group action.

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SIMPLE FINITE SUBGROUPS OF THE CREMONA GROUP OF RANK 3

YURI PROKHOROV

Abstract

We classify all finite simple subgroups of the Cremona group $\mathrm{Cr}_3(\mathbb{C})$.

1. Introduction

Let $\Bbbk$ be a field. The Cremona group $\mathrm{Cr}_d(\Bbbk)$ is the group of birational automorphisms of the projective space $\mathbb{P}^d_{\Bbbk}$, or, equivalently, the group of $\Bbbk$-automorphisms of the rational function field $\Bbbk(t_1, \dots, t_d)$. It is well-known that $\mathrm{Cr}_1(\Bbbk) = \mathrm{PGL}_2(\Bbbk)$. For $d \geq 2$, the structure of $\mathrm{Cr}_d(\Bbbk)$ and its subgroups is very complicated. For example, the classification of finite subgroups in $\mathrm{Cr}_2(\mathbb{C})$ is an old classical problem. Recently this classification almost has been completed by Dolgachev and Iskovskikh [DI06]. The following is a consequence of the list in [DI06].

Theorem 1.1 ([DI06]). *Let $G \subset \mathrm{Cr}_2(\mathbb{C})$ be a non-abelian simple finite subgroup. Then G is isomorphic to one of the following groups:*

$$\mathfrak{A}_5, \quad \mathfrak{A}_6, \quad \mathrm{PSL}_2(7). \tag{1.2}$$

However, the methods and results of [DI06] show that one cannot expect a reasonable classification of all finite subgroups of Cremona groups of higher rank. In this paper we restrict ourselves with the case of simple finite subgroups of $\mathrm{Cr}_3(\mathbb{C})$. Our main result is the following:

Theorem 1.3. *Let $G \subset \mathrm{Cr}_3(\mathbb{C})$ be a non-abelian simple finite subgroup. Then G is isomorphic to one of the following groups:*

$$\mathfrak{A}_5, \quad \mathfrak{A}_6, \quad \mathfrak{A}_7, \quad \mathrm{PSL}_2(7), \quad \mathrm{SL}_2(8), \quad \mathrm{PSp}_4(3). \tag{1.4}$$

All the possibilities occur.

In particular, we give the affirmative answer to a question of J.-P. Serre [Ser09, Question 6.0]: there are a lot of finite groups which do not admit any embeddings into $\mathrm{Cr}_3(\mathbb{C})$. More generally we classify simple finite subgroups in the group of birational automorphisms of an arbitrary three-dimensional rationally connected variety and in many cases we determine all birational models of the action:

Theorem 1.5. *Let X be a rationally connected threefold and let $G \subset \mathrm{Bir}(X)$ be a non-abelian simple finite subgroup. Then G is isomorphic either to $\mathrm{PSL}_2(11)$ or to one of the groups in the list* (1.4). *All the possibilities occur. Furthermore, if G does not*

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Kyoto Journal of Mathematics

Threefold extremal contractions of type (IA)

Shigefumi Mori and Yuri Prokhorov

Source: Kyoto J. Math. Volume 51, Number 2 (2011), 393-438.

Abstract

Let (X, C) be a germ of a threefold X with terminal singularities along an irreducible reduced complete curve C with a contraction $f : (X, C) \to (Z, o)$ such that $C = f^{-1}(o)_{\text{red}}$ and $-K_X$ is ample. Assume that a general member $F \in |-K_X|$ meets C only at one point P, and furthermore assume that (F, P) is Du Val of type A if index $(X, P) = 4$. We classify all such germs in terms of a general member $H \in |\mathcal{O}_X|$ containing C.

First Page: Show

Primary Subjects: 14J30, 14E, 14E30

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back to Table of Contents

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Prokhorov, Yuri

p-elementary subgroups of the Cremona group of rank 3. (English)

Faber, Carel (ed.) et al., Classification of algebraic varieties. Based on the conference on classification of varieties, Schiermonnikoog, Netherlands, May 2009. Zürich: European Mathematical Society (EMS). EMS Series of Congress Reports, 327-338 (2011). ISBN 978-3-03719-007-4/hbk

The author gives an upper bound for the rank of a p-elementary subgroup of the group $\mathrm{Bir}(X)$ of birational automorphisms of a rationally connected threefold defined over $\mathbb{C}$, i.e. subgroups of the form $G = (\mu_p)^s$, where μ_p is the cyclic group of order p. He also proves that the given bound is sharp when X is the projective space $\mathbb{P}^3$ and $p \geq 17$. More precisely, his main result asserts that for such a subgroup one obtains $s \leq 7$ (5; 4; 3, respectively) if $p = 2$ (3; 5, 7, 11 or 13; 17, respectively). As an important corollary it follows that if $\mathrm{Bir}(\mathbb{P}^N)$ is isomorphic to $\mathrm{Bir}(\mathbb{P}^3)$, then $N = 3$.
To obtain the main result the author first notices that one may suppose that G acts biregularly on X and then one may apply the G-equivariant minimal model program. This allows him to replace X with a G-equivariant Mori fiber space $f : \bar{X} \to Z$, in the category of projective normal varieties with at worst terminal singularities and where every G-stable Weil divisor is $\mathbb{Q}$-Cartier.
In the two cases where Z is not a point one considers the kernel G_0 of the natural homomorphism $G \to \mathrm{Aut}(Z)$. Then $G_1 := G/G_0$ acts transitively on Z and G_0 acts effectively on a general fiber of f. Since G_0 and G_1 are p-elementary with ranks s_0, s_1 such that $s = s_0 + s_1$, the main result follows, in this case, from the corresponding results for rational varieties of dimensions 1 and 2.
If Z is a point, i.e. $\bar{X}$ is a G-Fano variety, one has the most difficult and technical part of the proof. The rank of G is bounded in several steps depending on the genus, index and Picard number of $\bar{X}$, by taking into account the geometry of the pair $(\bar{X}, K_{\bar{X}})$, where $K_{\bar{X}}$ denotes a canonical divisor for $\bar{X}$.

Iván Pan (Montevideo)

Keywords : birational automorphisms; rank of a p-elementary subgroup
Classification :
$*$14E07 Birational automorphisms, Cremona group and generalizations

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Prokhorov, Yuri [Prokhorov, Yu. G.] (RS-MOSCMA-AL)
p-elementary subgroups of the Cremona group of rank 3. (English summary)
Classification of algebraic varieties, 327–338, *EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich,* 2011.

The author classifies possible p-elementary finite subgroups of the Cremona group of birational transformations of the three-dimensional projective space over an algebraically closed field of characteristic 0. Although, in the two-dimensional case, the classification of conjugacy classes of finite subgroups is more or less known [I. V. Dolgachev and V. A. Iskovskikh, in *Algebra, arithmetic, and geometry: in honor of Yu. I. Manin. Vol. I*, 443–548, Progr. Math., 269, Birkhäuser Boston, Inc., Boston, MA, 2009; MR2641179 (2011c:14032)], the similar problem in the three-dimensional case is wide open. The paper makes an important contribution in this direction. The main result asserts that the rank r of a p-elementary group of birational automorphisms of a rationally connected threefold is less than or equal to 7 if $p=2$, 5 if $p=3$, 4 if $p=5,7,11,13$, and 3 if $p \geq 17$. For any $p \geq 17$, this bound is attained for some subgroups of the Cremona group. To compare this result with the two-dimensional case, we have in this case $r \leq 2$ if $p>3$, $r \leq 4$ if $p=2$, and $r \leq 3$ if $p=3$ [A. Beauville, J. Algebra **314** (2007), no. 2, 553–564; MR2344578 (2008g:14019)].

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Holonomy of the Obata Connection on $SU(3)$

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A hypercomplex structure on a smooth manifold is a triple of integrable almost complex structures satisfying quaternionic relations. The Obata connection is the unique torsion-free connection that preserves each of the complex structures. The holonomy group of the Obata connection is contained in $GL(n, \mathbb{H})$. There is a well-known construction of hypercomplex structures on Lie groups due to Joyce. In this paper we show that the holonomy of the Obata connection on $SU(3)$ coincides with $GL(2, \mathbb{H})$.

1 Introduction

Consider a smooth manifold equipped with a triple of almost complex structures satisfying quaternionic relations. The manifold is called hypercomplex if these almost complex structures are integrable. Hypercomplex manifolds were defined by Boyer [3] and they have been much studied since then. There exist many examples of such manifolds including hyperkähler manifolds, nilmanifolds, Lie groups with hypercomplex structures, and others. Boyer also classified compact hypercomplex manifolds of real dimension 4. Homogeneous hypercomplex structures on Lie groups appeared in the context of string theory (see [10]) and then in the work of Joyce [6].

Each hypercomplex manifold is endowed with a torsion-free connection preserving all the complex structures, which is called the Obata connection.

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SPECIAL LAGRANGIAN FIBRATIONS ON THE FLAG VARIETY F^3

© **N. A. Tyurin***

We propose a construction of a Lagrangian torus fibration of the full flag variety in $\mathbb{C}^3$. In contrast to the classical fibration obtained from the Gelfand–Zeitlin system, the proposed fibration is special Lagrangian.

Keywords: flag variety, Lagrangian torus, pseudotoric structure, special Lagrangian fibration

1. Introduction

Lagrangian fibrations of symplectic manifolds are important for several applications. From the classical mechanics standpoint, if a phase space is fibered into Lagrangian submanifolds, then the motion of the corresponding system with a Hamiltonian function whose Hamiltonian vector field is parallel to the fibration can be described in terms of the fibers, and this can be done even in the case where certain fibers are singular.

One approach adopted in geometric quantization (GQ) is realized in the same setup. If a given phase space is fibered into Lagrangian submanifolds, then we take the so-called Bohr–Sommerfeld fibers, which form a discrete subset, and this subset generates the corresponding Hilbert space. We mention here that the notion of a Bohr–Sommerfeld Lagrangian submanifold is applicable to certain singular Lagrangian fibers. Homological mirror symmetry (HMS) can today be treated by considering a Lagrangian fibration and deriving a finite set of fibers satisfying certain conditions in terms of the Fukaya–Floer cohomology. Again, we could try to count holomorphic discs for certain singular tori, and it hence seems that this approach to HMS can be extended to the cases where the fibrations have singular fibers.

The ideal model situation for our considerations is provided by toric geometry [1]. If we work with a symplectic toric manifold, i.e., with the phase space of a completely integrable system, then there is a canonical Lagrangian fibration parameterized by a convex polytope, which leads to introducing the action–angle variables with a complete solution of the system. At the same time, the abovementioned GQ and HMS methods work beautifully in the toric case. The problem is that the set of toric manifolds is not so large and there exist other manifolds that do not have a toric structure for topological reasons but must be involved in the consideration.

For a compact symplectic toric manifold (M, ω), the canonical Lagrangian fibration is not completely regular. A dimensional reduction occurs, and smooth Lagrangian tori fill not the entire M but the complement $M \setminus D$, where D is a "symplectic divisor" whose homology class is Poincaré dual to the anticanonical class of M. The symplectic submanifold D of real codimension two consists of a number of irreducible components D_i, and each component as a symplectic manifold is fibered into Lagrangian tori that are isotropic in M. The general picture can be described in terms of the moment polytope P: each facet of it is the moment polytope of a smooth symplectic toric manifold whose real dimension is twice the dimension of the facet. In this setup, it is reasonable to call D the *boundary divisor*.

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Торы Чеканова и псевдоторические структуры

Н. А. Тюрин

Торы Чеканова доставляют примеры экзотических монотонных лагранжевых торов, не переводимых в стандартные торы симплектоморфизмами, в $\mathbb{R}^{2n}$, $\mathbb{CP}^1\times\dots\times\mathbb{CP}^1$ и некоторых проективных пространствах (см. [1]). Цепочка этих примеров вырастает из конструкции лагранжева тора $\Theta\subset\mathbb{R}^4$, представленного явно и неявно в ставших классическими работах [2] и [3]. С другой стороны, в работах [4], [5] было предложено обобщение понятия торической структуры на симплектических многообразиях, получившее название *псевдоторической структуры*. Так же, как и торическая, псевдоторическая структура порождает пространство лагранжевых слоений с общим гладким слоем, особые слои которых отличаются существованием на них сепаратрисных решений.

В настоящем кратком сообщении мы строим тор Чеканова $\Theta^k\subset\mathbb{R}^{2k-2}$, определенный в [1], как гладкий лагранжев тор, порождаемый псевдоторической структурой. Отсюда вытекает возможность построения экзотических монотонных лагранжевых торов типа Чеканова на торических многообразиях Фано.

Рассмотрим $\mathbb{R}^{2k+2}$ со стандартными комплексной и симплектической структурами, т.е. $\mathbb{C}^{k+1}$. Зафиксируем координаты $(z_1,\dots,z_{k+1})$; тогда имеется отображение $\psi\colon\mathbb{C}^{k+1}\to\mathbb{C}$, задаваемое формулой

$$(z_1,\dots,z_{k+1})\mapsto a=z_1\times\dots\times z_{k+1}\in\mathbb{C}_a.$$

Слои ψ – гиперповерхности в $\mathbb{C}^{k+1}$, гладкие за исключением слоя над нулем. Гамильтоново действие тора T^{k+1} может быть выбрано так, что имеется подтор T^k_0, действие которого сохраняет слои ψ. А именно, этот подтор выделен условием

$$\sum_{i=1}^{k+1}\alpha_i=0,$$

где α_i – веса. Отображения моментов для гамильтонова действия T^k_0 даются средними значениями $F_i=\langle A_i\psi;\psi\rangle$, $\psi\in\mathbb{C}^{k+1}$, самосопряженных операторов $A_i=\operatorname{diag}(\lambda_1,\dots,\lambda_{k+1})$, где $\lambda_i=1$, $\lambda_{i+1}=-1$ и $\lambda_j=0$ для остальных индексов. Тогда нетрудно видеть, что $\{F_1,\dots,F_k\}$ – коммутирующий набор на $\mathbb{C}^{k+1}$, ограничение которого на каждый гладкий слой ψ есть вполне интегрируемая система (см. [6]). Структура такого типа называется *псевдоторической структурой* (детальное определение см. в [5]).

Если зафиксировать набор $(c_1,\dots,c_k)$ некритических значений для ограничений $F^a_1,\dots,F^a_k$ на произвольный гладкий слой N_a (т.е. $-1<c_i<1$), то совместное множество уровня $S^a_{(c_1,\dots,c_k)}=\{F^a_i=c_i\}\subset N_a$ – гладкий лагранжев тор для каждого $a\neq0$. Тогда выбор произвольной гладкой петли $\gamma\subset\mathbb{C}^*_a=\mathbb{C}_a\setminus\{0\}$ определяет гладкий лагранжев тор в $\mathbb{C}^{k+1}$:

$$T^{k+1}=S_{\gamma,(c_1,\dots,c_k)}=\bigcup_{a\in\gamma}S^a_{(c_1,\dots,c_n)}.$$

Различаются два случая: стандартный тип, когда γ нестягиваема в $\mathbb{C}^*_a$; тип Чеканова, когда γ стягиваема в $\mathbb{C}^*_a$. Названия типов объясняются следующим утверждением.

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Communications in
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A Finite Analog of the AGT Relation I: Finite W-Algebras and Quasimaps' Spaces

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Abstract: Recently Alday, Gaiotto and Tachikawa [2] proposed a conjecture relating 4-dimensional super-symmetric gauge theory for a gauge group G with certain 2-dimensional conformal field theory. This conjecture implies the existence of certain structures on the (equivariant) intersection cohomology of the Uhlenbeck partial compactification of the moduli space of framed G-bundles on $\mathbb{P}^2$. More precisely, it predicts the existence of an action of the corresponding W-algebra on the above cohomology, satisfying certain properties.

We propose a "finite analog" of the (above corollary of the) AGT conjecture. Namely, we replace the Uhlenbeck space with the space of *based quasi-maps* from $\mathbb{P}^1$ to any partial flag variety G/P of G and conjecture that its equivariant intersection cohomology carries an action of the finite W-algebra $U(\mathfrak{g}, e)$ associated with the principal nilpotent element in the Lie algebra of the Levi subgroup of P; this action is expected to satisfy some list of natural properties. This conjecture generalizes the main result of [5] when P is the Borel subgroup. We prove our conjecture for $G = GL(N)$, using the works of Brundan and Kleshchev interpreting the algebra $U(\mathfrak{g}, e)$ in terms of certain shifted Yangians.

1. Introduction

1.1. The setup. Let G be a semi-simple simply connected complex algebraic group (or, more generally, a connected reductive group whose derived group $[G, G]$ is simply connected) and let P be a parabolic subgroup of G. We shall denote by L the corresponding Levi factor. Let B be a Borel subgroup of G contained in P and containing a maximal torus T of G. We shall also denote by Λ the coweight lattice of G (which is the same as the lattice of cocharacters of T); it has a quotient lattice $\Lambda_{G,P} = \mathrm{Hom}(\mathbb{C}^*, L/[L, L])$, which can also be regarded as the lattice of characters of the center $Z(\check{L})$ of the Langlands dual group $\check{L}$. Note that $\Lambda_{G,B} = \Lambda$. The lattice $\Lambda_{G,P}$ contains the canonical sub-semi-group $\Lambda^+_{G,P}$ spanned by the images of positive coroots of G.

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Character D-modules via Drinfeld center of Harish-Chandra bimodules

Roman Bezrukavnikov · Michael Finkelberg · Victor Ostrik

To Tony Joseph, with best wishes on his anniversary

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Abstract The category of character D-modules is realized as Drinfeld center of the abelian monoidal category of Harish-Chandra bimodules. Tensor product of Harish-Chandra bimodules is related to convolution of D-modules via the long intertwining functor (Radon transform) by a result of Beilinson and Ginzburg (Represent. Theory 3, 1–31, 1999). Exactness property of the long intertwining functor on a cell subquotient of the Harish-Chandra bimodules category shows that the truncated convolution category of Lusztig (Adv. Math. 129, 85–98, 1997) can be realized as a subquotient of the category of Harish-Chandra bimodules. Together with the description of the truncated convolution category (Bezrukavnikov et al. in Isr. J. Math. 170, 207–234, 2009) this allows us to derive (under a mild technical assumption) a classification of irreducible character sheaves over $\mathbb{C}$ obtained by Lusztig by a different method.

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DYNAMICAL WEYL GROUPS AND EQUIVARIANT COHOMOLOGY OF TRANSVERSAL SLICES ON AFFINE GRASSMANNIANS

ALEXANDER BRAVERMAN AND MICHAEL FINKELBERG

ABSTRACT. Let G be a reductive group and let $\check{G}$ be its Langlands dual. We give an interpretation of the dynamical Weyl group of $\check{G}$ defined in [5] in terms of the geometry of the affine Grassmannian Gr of G. In this interpretation the dynamical parameters of [5] correspond to equivariant parameters with respect to certain natural torus acting on Gr. We also present a conjectural generalization of our results to the case of affine Kac-Moody groups.

1. Introduction and statements of the results

1.1. Notations and overview. We mainly follow the notations of [3]. So $G \supset B \supset T$ is a reductive complex algebraic group with a Borel subgroup, and a Cartan subgroup. Let $\check{G} \supset \check{T}$ denote the Langlands dual group with its Cartan torus. Let $\mathcal{K} = \mathbb{C}((t))$, $\mathcal{O} = \mathbb{C}[[t]]$. The affine Grassmannian $\mathrm{Gr} = \mathrm{Gr}_G = G(\mathcal{K})/G(\mathcal{O})$ carries the category $\mathrm{Perv}_{G(\mathcal{O})}(\mathrm{Gr})$ of $G(\mathcal{O})$-equivariant perverse constructible sheaves. It is equipped with the convolution monoidal structure, and is tensor equivalent to the tensor category $\mathrm{Rep}(\check{G})$ of representations of $\check{G}$ (see [8], [6], [10], [2]). We denote by $\tilde{S}: \mathrm{Rep}(\check{G}) \to \mathrm{Perv}_{G(\mathcal{O})}(\mathrm{Gr})$ the geometric Satake isomorphism functor, and by $S: \mathrm{Rep}(\check{G}) \to \mathrm{Perv}_{G(\mathcal{O})\rtimes\mathbb{C}^*}(\mathrm{Gr})$ its extension to the monoidal category of $G(\mathcal{O})\rtimes\mathbb{C}^*$-equivariant perverse constructible sheaves.

The Lie algebras of $\check{G} \supset \check{T}$ are denoted by $\check{\mathfrak{g}} \supset \check{\mathfrak{t}}$. We have $\check{\mathfrak{t}} = \mathfrak{t}^*$ canonically. The Weyl group of $G, \check{G}$ is denoted by W. Let $\Lambda = \Lambda_G$ denote the coweight lattice of T which is the same as the weight lattice of $\check{T}$. The choice of Borel subgroup $B \subset G$ defines the cone $\Lambda_G^+ \subset \Lambda_G$ of dominant weights of $\check{T}$.

The lattice $\Lambda = \Lambda_G$ is identified with the quotient $T(\mathcal{K})/T(\mathcal{O})$. For $\lambda \in \Lambda$ we denote by t^λ any lift of λ to $T(\mathcal{K})$. Its image in $\mathrm{Gr}_G = G(\mathcal{K})/G(\mathcal{O})$ is independent of the choice of a lift, and will be also denoted by t^λ, or sometimes just $\lambda \in \mathrm{Gr}_G$. Moreover, we will keep the same name for the closed embedding $\lambda \hookrightarrow \mathrm{Gr}$. Let Gr^λ denote the $G(\mathcal{O})$-orbit of λ, and let $\overline{\mathrm{Gr}}^\lambda \subset \mathrm{Gr}$ denote the closure of Gr^λ. It is well known that $\mathrm{Gr} = \bigcup_{\lambda\in\Lambda} \mathrm{Gr}^\lambda$, and that $\mathrm{Gr}^\lambda = \mathrm{Gr}^\mu$ iff λ and μ lie in the same W-orbit on Λ. In particular, $\mathrm{Gr} = \bigsqcup_{\lambda\in\Lambda^+} \mathrm{Gr}^\lambda$.

It is known that as a byproduct of the existence of S (or $\widetilde{S}$) one can get a geometric construction of finite-dimensional representations of $\check{G}$. It is not difficult to note

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Gelfand–Tsetlin algebras and cohomology rings of Laumon spaces

Boris Feigin · Michael Finkelberg · Igor Frenkel · Leonid Rybnikov

To the memory of Izrail Moiseevich Gelfand

Published online: 25 November 2010

Abstract Laumon moduli spaces are certain smooth closures of the moduli spaces of maps from the projective line to the flag variety of GL_n. We calculate the equivariant cohomology rings of the Laumon moduli spaces in terms of Gelfand–Tsetlin subalgebra of $U(gl_n)$ and formulate a conjectural answer for the small quantum cohomology rings in terms of certain commutative shift of argument subalgebras of $U(gl_n)$.

Mathematics Subject Classification (2000) 20C99

1 Introduction

1.1 Cohomology of Laumon spaces

The moduli spaces $\mathcal{Q}_{\underline{d}}$ were introduced by G. Laumon in [15] and [14]. They are certain compactifications of the moduli spaces of degree $\underline{d}$ maps from $\mathbb{P}^1$ to the flag

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Yangians and cohomology rings of Laumon spaces

Boris Feigin · Michael Finkelberg ·
Andrei Negut · Leonid Rybnikov

To our friend Sasha Shen on his 50th birthday

Published online: 22 April 2011

Abstract Laumon moduli spaces are certain smooth closures of the moduli spaces of maps from the projective line to the flag variety of GL_n. We construct the action of the Yangian of $\mathfrak{sl}_n$ in the cohomology of Laumon spaces by certain natural correspondences. We construct the action of the affine Yangian (two-parametric deformation of the universal enveloping algebra of the universal central extension of $\mathfrak{sl}_n[s^{\pm 1}, t]$) in the cohomology of the affine version of Laumon spaces. We compute the matrix coefficients of the generators of the affine Yangian in the fixed point basis of cohomology. This basis is an affine analog of the Gelfand-Tsetlin basis. The affine analog of the Gelfand-Tsetlin algebra surjects onto the equivariant cohomology rings of the affine Laumon spaces. The cohomology ring of the moduli space $\mathfrak{M}_{n,d}$ of torsion free sheaves on the plane, of rank n and second Chern class d, trivialized at infinity, is

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On exceptional quotient singularities

Ivan Cheltsov and Constantin Shramov

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Abstract

We study exceptional quotient singularities. In particular, we prove an exceptionality criterion in terms of the α-invariant of Tian, and utilize it to classify four-dimensional and five-dimensional exceptional quotient singularities.

Keywords

quotient singularities, log canonical threshold, alpha invariant, Fano, group, Kähler–Einstein metric

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Local properties of J-complex curves in Lipschitz-continuous structures

S. Ivashkovich · V. Shevchishin

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Abstract We prove the existeuce of primitive curves and positivity of intersections of J-complex curves for Lipschitz-continuous almost complex structures. These results are deduced from the Comparison Theorem for J-holomorphic maps in Lipschitz structures, previously known for J of class $C^{1,Lip}$. We also give the optimal regularity of curves in Lipschitz structures. It occurs to be $C^{1,LnLip}$, i.e. the first derivatives of a J-complex curve for Lipschitz J are Log-Lipschitz-continuous. A simple example that nothing better can be achieved is given. Further we prove the Genus Formula for J-complex curves and determine their principal Puiseux exponents (all this for Lipschitz-continuous J-s).

Keywords Almost complex structure · Pseudoholomorphic curve · Cusp · Genus Formula · Puiseux exponents

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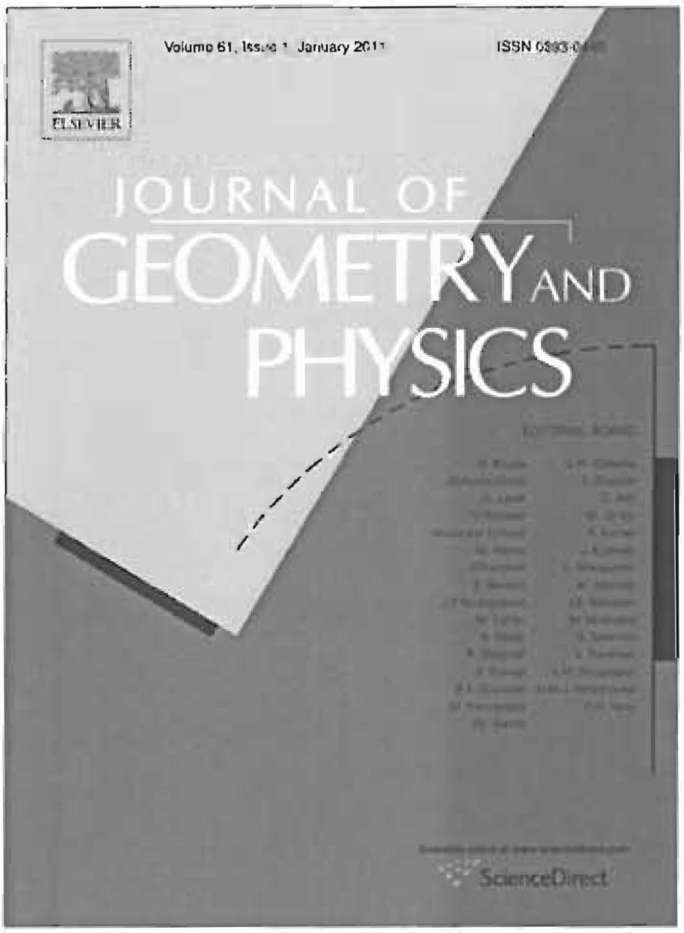
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Two-dimensional superintegrable metrics with one linear and one cubic integral

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ABSTRACT

We describe all local Riemannian metrics on surfaces whose geodesic flows are superintegrable with one integral linear in momenta and one integral cubic in momenta.

We also show that some of these metrics can be extended to S^2. This gives us new examples of Hamiltonian systems on the sphere with integrals of degree three in momenta, and the first examples of superintegrable metrics of nonconstant curvature on a closed surface.

1. Introduction

1.1. Definitions and statement of the problem

Let M^2 be a surface (i.e., 2-dimensional real manifold) equipped with a Riemannian metric $g = (g_{ij})$. The *geodesic flow* of the metric g is the Hamiltonian system on the cotangent bundle T^*M^2 with the Hamiltonian $H := \frac{1}{2}g^{ij}p_ip_j$, where $(x, y) = (x_1, x_2)$ is a local coordinate system on M^2, and $(p_x, p_y) = (p_1, p_2)$ are the correspondent *momenta*, i.e., the dual coordinates on T^*M^2.

We say that a function $F : T^*M^2 \to \mathbb{R}$ is an *integral* of the geodesic flow of g, if $\{F, H\} = 0$, where $\{,\}$ is the canonical Poisson bracket on T^*M^2. We say that the integral is *polynomial in momenta of degree d*, if in every local coordinate system

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